The Feldtkeller equation describes energy relations in general. It can be derived from what are called the unitary properties of the S-matrix for lossless networks. For lossless networks the following is true:

$$\left[S^*\right]^T [S] = [I]$$

$$\begin{bmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{bmatrix}^* \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad [1]$$

From [1] we can deduce:

$$|S_{11}|^2 + |S_{21}|^2 = 1 \qquad (2) \qquad |S_{12}|^2 + |S_{22}|^2 = 1 \qquad (3)$$

$$S_{11}{}^* S_{12} + S_{21}{}^* S_{22} = 0 \qquad (4)$$

$$S_{12} = -\frac{S_{21}{}^* S_{22}}{S_{11}{}^*} \qquad (4a)$$

$$\frac{|S_{21}|^2 |S_{22}|^2}{|S_{11}|^2} + |S_{22}|^2 = 1$$

Substituting (4a) into (3) gives $|S_{21}|^2 |S_{22}|^2 + |S_{11}|^2 |S_{22}|^2 = |S_{11}|^2$

$$|S_{21}|^2 |S_{22}|^2 = |S_{11}|^2 \left(1 - |S_{22}|^2\right) \quad \text{substitute from (3)}$$

$$|S_{21}|^2 |S_{22}|^2 = |S_{11}|^2 |S_{12}|^2$$

This last expression is true if and only if

$$|S_{21}| = |S_{12}| \qquad and \qquad |S_{11}| = |S_{22}|$$

Returning to equation (2):

$$|S_{12}|^2 + |S_{22}|^2 = |S_{21}|^2 + |S_{22}|^2 = 1 \qquad \text{due to } S_{12} = S_{21}$$

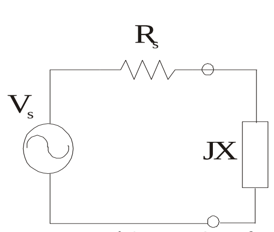
Substituting from (3) and using the property that $S_{21} = S_{12}$ gives:

$$|S_{21}|^2 + \left(1 - |S_{21}|^2\right) = 1$$

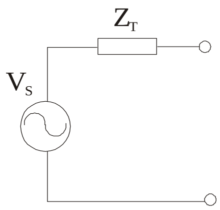
$$kT|S_{21}|^2 + kT\left(1 - |S_{21}|^2\right) = kT$$

The result of this is that for a lossless network the amount of noise power delivered to the load is a constant kT watts, regardless if the noise is attenuated by a lossless (reactance-only) filter.

For a passive element the noise figure is given by: $F = \dfrac{1}{G_A}$ $\quad$ *where* $G_A =$ available gain



Looking at a shunt reactance (to the left) with the intent of seeing the NF impacts of a lossless transmission line, a series of Thevenin and Norton transformations gives the following results:



After the transformations we have:

$$V'_s = \frac{jV_g R_g X}{R_g\left(R_g + jX\right)} \qquad Z_T = \frac{jR_g^2 X + R_g X^2}{R_g^2 + X^2}$$

The noise power available from the source is:

$$P_{noise} = \frac{V_g^2}{4R_g} = \frac{\dfrac{V_g{}^2}{R_g{}^2}\dfrac{\left(R_g X\right)^2}{\left(R_g{}^2 + X^2\right)}}{4\dfrac{R_g X^2}{R_g{}^2 + X^2}} = \frac{V_g{}^2}{4R_g}$$

The noise figure for a shunt reactive network (zero resistive loss) is 1.0.

It appears for lossless networks of passive elements, transmission efficiency is impacted by $\left|S_{21}\right|^2$, i.e. impedance mismatches, however the noise figure is not impacted.

