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The power delivered to any device as a function of time is given by the product of the instantaneous voltage across the device and the instantaneous current through it.

For an inductor the following is true:

$$p = v_L\, i_L \;= L\, i_L \frac{di}{dt} = \frac{1}{L} v_L \int_{-\infty}^{t} v(t)\, dt$$

For a single series R-L circuit fed by a voltage unit step $V_o$ u(t) the following equations can be written:

$$-V_o\, u(t) + L\frac{di}{dt} + i\,R = 0$$

$$V_o\, u(t) = L\frac{di}{dt} + i\,R$$

$$\frac{V_o}{s} = I(s)\left[\, s\,L + R\right]$$

$$I(s) = \frac{V_o}{s\left[\, sL + R\,\right]}$$

Using partial fraction decomposition, this expression for I( s ) can be written to facilitate determining the time expression.

$$I(s) = \frac{V_o}{s\left[\, sL + R\,\right]} = \frac{A}{s} + \frac{B}{sL + R}$$

$$A = \frac{V_o}{R} \qquad B = -\,A\,L = -V_o\frac{L}{R}$$

Putting all these expressions together yields the final expression for the current

$$I(s) = \frac{V_o / R}{s} + \frac{-A / R}{s + R / L} \quad = \quad \frac{V_o}{R}\left[1 - e^{-\frac{Rt}{L}}\right] u(t)$$

The power delivered by the source or absorbed by the passive network is then:

$$p = vi = \frac{V_o^{\,2}}{R}\left[1 - e^{-\frac{Rt}{L}}\right]^2 u(t) \quad [\,1\,]$$

Power delivered to the resistor is

$$p_R = v\,i = i^2\,R = \frac{V_o^{\,2}}{R}\left[1 - e^{-\frac{Rt}{L}}\right]^2 u(t) \qquad [\,2\,]$$

The power delivered to the inductor is slightly more complex to determine.

$$v_L = L\frac{di}{dt}$$

Taking the derivative of the current w.r.t. time gives:

$$\frac{di(t)}{dt} = \frac{d}{dt}\left[\left(\frac{V_o}{R} - \frac{V_o}{R}e^{-\frac{Rt}{L}}\right)u(t)\right] = \frac{V_o}{R}\delta(t) - \frac{V_o}{R}\left[\, e^{-\frac{Rt}{L}}\,\delta(t) - \frac{R}{L}e^{-\frac{Rt}{L}}\, u(t)\right]$$

Combining things and noticing the cancellation of the Dirac delta terms at t = 0 gives the following for the voltage across the inductor:

$$v_L = \frac{V_o}{L} e^{-\frac{Rt}{L}} u(t) \qquad [3]$$

Now, for the power developed in the inductor, we multiply the time-domain voltage and current together to obtain

$$p_L = v_L\, i = L\frac{di}{dt} i_L = L\left[\frac{V_o}{L} e^{-\frac{Rt}{L}} u(t)\right]\frac{V_o}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

[ 4, 5 ]

$$p_L = \frac{V_o^{\,2}}{R} e^{-\frac{Rt}{L}}\left(1 - e^{-\frac{Rt}{L}}\right) u(t)$$

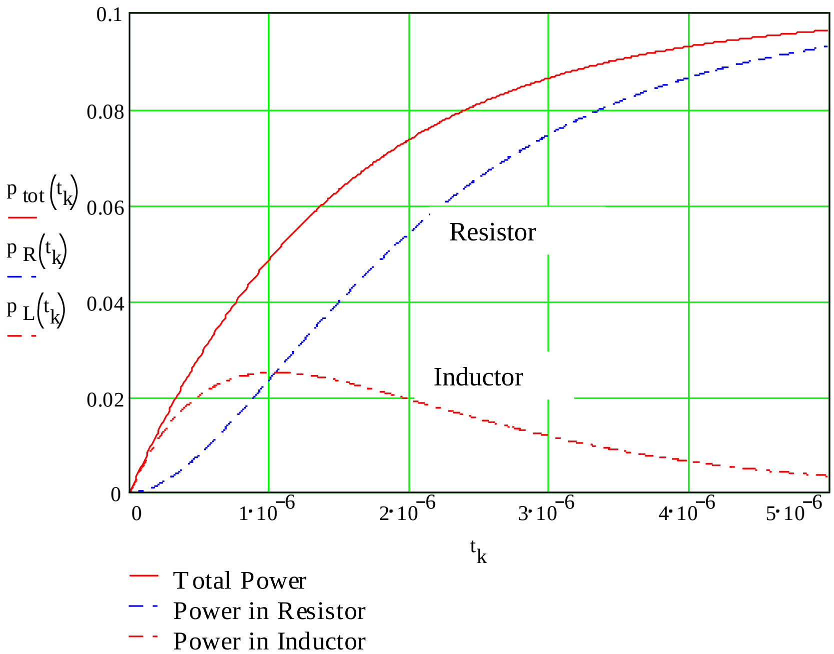


*Figure 1*

As time progresses the total power in the circuit increases up to the maximum possible of 0.1 Watt. This is set by the value of the resistor: 10 ohms. The L and C values used were 4.7 H and 15 F. One time constant for the values used is equal to 1.5 sec.

The input stimulus for this problem was a unit step function of magnitude 1.0 volts.

Let us now replace the input unit step function with a sinusoidal source of same magnitude $V_o$. Applying Laplace-type analysis gives us a similar expression to consider, however with increased complexity.

$$-V_o \cos(\omega_s t) + L\frac{di}{dt} + i(t) = 0 \qquad \Rightarrow \qquad \frac{V_o\, s}{s^2 + \omega_s^{\,2}} = I(s)(sL + R)$$

 

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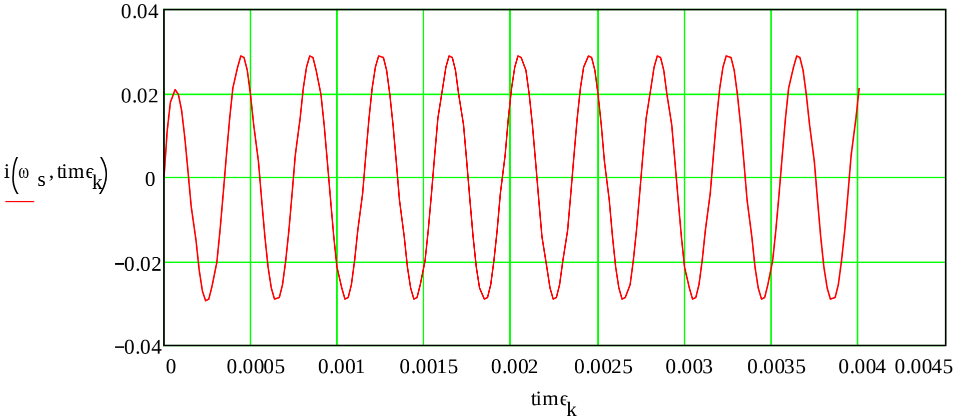
**Reference:** **Author:** Jeff Crawford

$$\frac{V_o\, s}{\left(s^2+\omega_s{}^2\right)\left(s\,L+R\right)}=\frac{A\,s+B}{s^2+\omega_s{}^2}+\frac{C}{s\,L+R}\;\Rightarrow\qquad A=\frac{V_o R}{\left(\omega_s\,L\right)^2+R^2}\qquad B=\frac{V_o L\,\omega_s{}^2}{\left(\omega_s\,L\right)^2+R^2}\qquad C=\frac{L\left(L-RV_o\right)}{\left(\omega_s\,L\right)^2+R^2}$$

$$I(\,s\,)=\frac{V_o\, s}{\left(s^2+\omega_s{}^2\right)\left(s\,L+R\right)}\qquad [\,6\,]$$




Taking the inverse Laplace transform of this last expression gives the current I( t ) which is graphed as a function of time in Figure 2. As is seen in Figure 2, the steady-state current takes a small period of time to build up after the transient response.

$$i(\,t\,)=A\cos(\,\omega_s\,t\,)+\frac{B}{\omega_s}\sin(\,\omega_s\,t\,)+\frac{C}{L}e^{-\frac{Rt}{L}}\qquad [\,7\,]$$

The power delivered to the resistor is straight forward with the expression for current in [7]: I( t )$^2$ R. Using the following expression for di/dt, the voltage across the inductor L can be determined, then multiplied by i( t ) to determine the instantaneous power delivered to the inductor as well.

$$v_L=L\frac{di}{dt}=L\left[-A\omega_s\sin(\omega_s t)+\frac{B\omega_s}{\omega_s}\cos(\omega_s t)+\frac{RC}{L^2}e^{-\frac{Rt}{L}}\right]\qquad [\,8\,]$$

The power delivered to the inductor is then $v_L$( t ) i( t ).

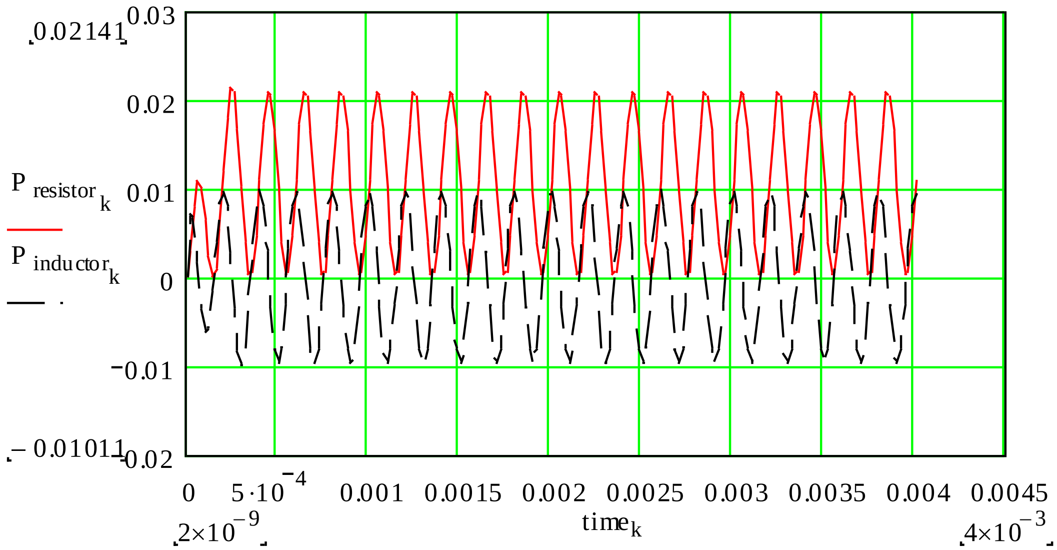



Figure 3

Adding the individual powers developed in the resistor and inductor gives the same total power delivered to the circuit, found by multiplying v( t ) by i( t ).

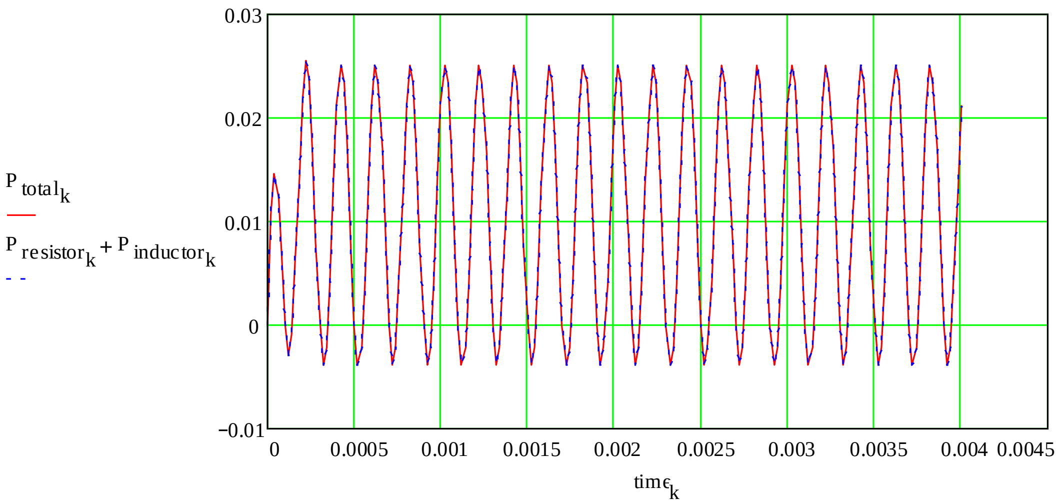



*Figure 4*
Total Power Equals the Summation of the Individual Powers

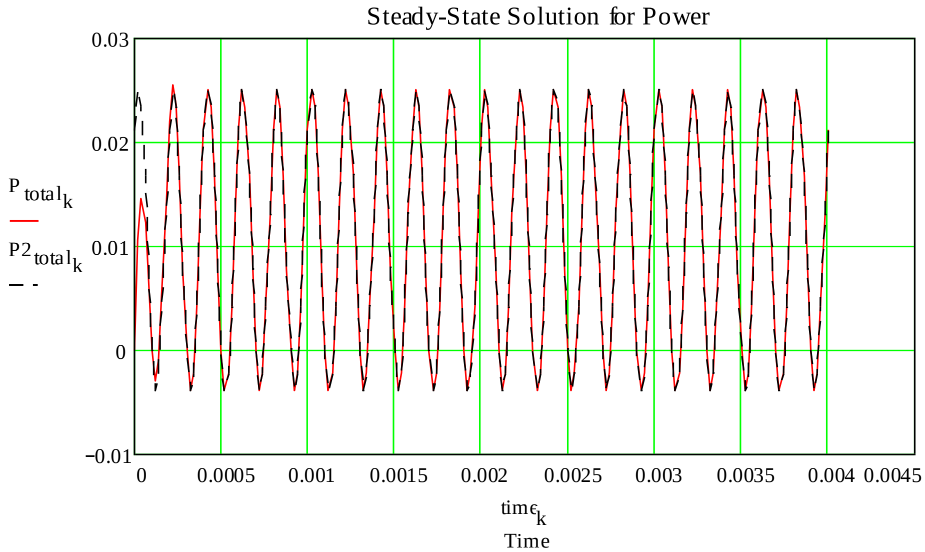



*Figure 5*
Steady-State Power Lacks Initial Transient Response Found in the Complete Solution