Resonance is the condition when the input impedance of the network is purely resistive. A network is said to be in resonance when the voltage and current are in phase.

A maximum amplitude response is produced in the network when it is in the resonant condition or almost in the resonant condition.

The admittance of a parallel R-L-C circuit is:

$$\mathbf{Y} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

Resonance occurs when the reactive component is zero: $\omega C - \dfrac{1}{\omega L} = 0$

The resonant frequency is: $\omega_o = \dfrac{1}{\sqrt{LC}}$

The pole-zero configuration of the admittance function can also be used to considerable advantage here:

$$\mathbf{Y}(s) = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) = \frac{1}{R} + \frac{1}{sL} + sC$$

$$\mathbf{Y}(s) = C\,\frac{s^2 + s/RC + 1/LC}{s} = \frac{(s + \alpha - j\omega_d)(s + \alpha + j\omega_d)}{s}$$

α is the exponential damping coefficient: $\alpha = \dfrac{1}{2RC}$

ω_d is the natural resonant frequency (not the resonant frequency ω_o) $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

It is apparent that the distance from the origin of the s-plane to one of the admittance zeros is numerically equal to ω_o.

The intersection of this arc and the positive $j\omega$ axis locates the point $s = j\omega_o$. ω_o is slightly greater than the natural frequency ω_d.

For an input current source the maximum value of the response across the parallel network is indicated as R times the amplitude of the source current, implying the maximum magnitude of the circuit impedance is R. The maximum is shown to occur exactly at the resonant frequency ω_o.

In a parallel R-L-C network the maximum impedance level possible is R.

At resonance the current through L and C can be calculated: $I = \dfrac{V}{Z}$ where V = IR

$$I_{L,o} = \frac{IR}{j\omega_o L} \qquad I_{C,o} = IR\, j\omega_o C$$

Since at resonance: $\dfrac{1}{\omega_o C} = \omega_o L$, $I_{L,o} = -I_{C,o} = j\omega_o RCI$

The height of the response curve in a parallel RLC circuit depends only upon R.

The sharpness of the response curve of any resonant circuit is determined by the maximum amount of energy that can be stored in the circuit, compared with the energy that is lost during one complete period of the response.

$$Q = 2\pi \frac{\text{Max Energy Stored}}{\text{Total Energy Lost per Period}} \qquad Q = \frac{2\pi\left[W_L(t)+W_C(t)\right]_{\max}}{P_R T}$$

Parallel RLC

$$i(t) = I_m \cos(\omega_o t)$$

At resonance, $v(t) = i(t)R = I_m R\cos(\omega_o t)$

Energy stored in the *capacitor* is

$$P_C = i_C v_C$$
$$= C\, v_C \frac{d\, v_C}{dt}$$
$$Energy = \int_0^T P_C dt = C\int_0^T v_C\left(\frac{d\, v_C}{dt}\right) dt = \frac{1}{2} C\, v_C^2$$

Therefore:

$$W_C = \frac{1}{2} C R^2 I_m^2 \cos(\omega_o t)$$

At resonance

$$i_C = C\frac{dv_C}{dt} = -C I_m R \sin(\omega_o t)\,\omega_o$$
$$= -(\omega_o RC) I_m \sin(\omega_o t)$$
$$= -Q_P I_m \sin(\omega_o t)$$

For a parallel R-L-C network at resonance:

$$|I_C| = |I_L| = Q_P I_m$$

In a similar manner the energy stored in the inductor is:

$$W_L = \frac{1}{2} L i^2 \qquad v_L = L\frac{di(t)}{dt} \qquad i = \frac{1}{L}\int_0^T v_L(t)\,dt$$

$$W_L = \frac{L}{2}\left(\int_0^T v_L(t)\,dt\right)^2 = \frac{L}{2}\left(\frac{1}{L}\int_0^t I_m R\cos(\omega_o t)\,dt\right)^2$$

$$= \frac{L}{2}\left(\frac{1}{L}\frac{I_m R}{\omega_o}\sin(\omega_o t)\Big|_0^t\right)^2 = \frac{L}{2}\left(\frac{I_m R}{L\omega_o}\sin(\omega_o t)\right)^2$$

$$= \frac{L}{2}\frac{I_m^2 R^2}{L^2\omega_o^2}\sin^2(\omega_o t)$$

$$W_L = \frac{1}{2}I_m^2 R^2 C\sin^2(\omega_o t)$$

So
$$W_L + W_C = \frac{I_m^2 R^2 C}{2}\left(\cos^2(\omega_o t) + \sin^2(\omega_o t)\right)$$
$$= \frac{I_m^2 R^2 C}{2} = \text{constant}$$

The energy lost in the resistor in one period is

$$T\int_0^t I_m^2 \cos^2(\omega_o t) R\ dt$$

$$P_R T = \frac{I_m^2 R}{2}T = \frac{I_m^2 R}{2\,f_o}$$

$$Q_o = \frac{I_m^2 R^2 C/2}{I_m^2 R/2\,f_o} = 2\pi f_o RC = \omega_o RC$$